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Thermochimica Acta 320 (1998) 253–263

thermochimica
acta

Molecular thermodynamic model for equilibria in solution

I. Reacting and non-reacting ensembles

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Received 14 January 1998; accepted 1 July 1998

Abstract

The partition functions of solution thermodynamics are mathematical representations of the properties of molecular ensembles statistically distributed according to specific characteristics. The ensembles are classified as *non-reacting* or *reacting*. The *non-reacting* ensembles are characterized by one mean enthalpy level with dispersion around the mean. The *reacting* ensembles are characterized by two or more distinctly separated enthalpy levels over which the different species are variably distributed, depending on concentration and/or temperature.

The *non-reacting* ensembles can be distinguished into *microcanonical*, *thermal*, *osmotic*, *thermo-osmotic* ensembles, depending on the type of exchange with the surroundings which is connected to the fluctuations of the ensemble variables. The reacting ensembles can be distinguished into *thermal*, *osmotic*, *thermo-osmotic*, *electrochemical*, *electro-osmotic*, *electro-thermal*, *electro-thermo-osmotic* ensembles, depending on the type of reaction and of exchange with the surroundings. © 1998 Elsevier Science B.V.

Keywords: Chemical equilibria; Statistical mechanics; Theory

1. Introduction

We have been applying [1–13] for the last 10 years, the binding polynomials [14,15] to the study of the equilibria in solution. The binding polynomials are related to the distributions of the concentrations of the species. As such, they are related to the probability of finding the species in the unit volume of the solution. The use of probabilities to describe the properties of a system is typical of statistical thermodynamics. Following Poland [15], we have identified the binding polynomials as special cases of partition functions [10], as they are used in statistical thermodynamics. Recently, Ben-Naim [16] has shown how the properties of chemical solutions can be described on a statistical thermodynamic basis. The statistical description is made on the assumption of certain types of molecular ensembles as cornerstones of the whole statistical framework.

The purpose of this investigation is to demonstrate that the definitions of statistical distributions of molecular ensembles *reacting* or *non-reacting* in solution are consistent with the basic foundations of statistical thermodynamics. Thus, it will be possible to show how equilibrium constant, heat of reaction, heat capacity, buffer capacity, binding function, binding polynomial, formation function, saturation fraction,

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PII S0040-6031(98)00475-4

correlation function, etc. are the experimental macroscopic features of transformations undergone by the molecular distributions described by the model ensembles.

2. Ensembles

An ensemble is considered as a collection of many cells which are replicas of the whole thermodynamic system. *Reacting* or *non-reacting* is the internal requisite of a system necessary to define the ensemble. The definition of a *reacting* or *non-reacting* system is analogous to the definition of an interacting or non-interacting system. Even a self associating pure compound can become reacting, e.g. a structured liquid.

The *non-reacting* ensembles (*nre*) are characterized by one mean enthalpy level and by dispersion around the mean due to fluctuations in concentration and/or temperature. The *reacting* ensembles (*re*) are characterized by the existence of two or more distinctly separated mean enthalpy levels over which the different species are variably distributed on dependence of concentration and/or temperature.

These definitions of ensembles have been assumed by us in order to coherently explain the properties of the solutions. The specification of the external conditions, e.g. '*open*' or '*closed*' to exchanges of heat and matter are not sufficient in themselves to define the statistical ensemble. The specification concerning the intrinsic properties of *reacting* or *non-reacting* ensembles is an implementation of the usual conventions adopted in statistical thermodynamics, where the properties of an ensemble concern only the boundary conditions of the system. Usually, a microcanonical ensemble is representative of an '*isolated*' system, which does not exchange either heat or matter with the surroundings. A canonical ensemble is representative of a '*closed*' system which exchanges heat but not matter with the surroundings. A grand canonical ensemble is associated to an '*open*' system which exchanges both, heat and matter with the surroundings. The type of exchange with the surroundings, however, is insufficient by itself, in some cases, to describe the behaviour of the thermodynamic system if not associated to the definition of *reacting* or *non-reacting* system. Specifically, a redistribution of species among levels takes place in a closed (canonical) system if it is *reacting* and, conversely, there is no redistribution of species among levels in an open (grand canonical) system if it is *non-reacting*. These changes in the distribution of populations are possible if, and only if, outstanding enthalpy differences between species (or levels) exists inside the system.

The type of external conditions or exchange with the surroundings can be employed to distinguish the types of *nre* or *re* from one another. The *nre* can be distinguished into *microcanonical*, *thermal*, *osmotic*, *thermo-osmotic* ensembles depending on the type of exchange with the surroundings. The *microcanonical* ensemble is identical with the classical microcanonical ensemble closed to heat and matter. The *thermal nre* is closed to exchange of matter and, therefore, corresponds to a classical canonical ensemble, whereas the *osmotic nre* is open and corresponds to a classical grand canonical ensemble. The *re* can be distinguished into *thermal*, *osmotic*, *thermo-osmotic*, *electrochemical*, *electro-osmotic*, *electro-thermal*, *electro-thermo-osmotic* ensembles depending on the type of exchange with the surroundings. Again, the *thermal re* ('*closed*') corresponds to a canonical ensemble, whereas all the other *re* are '*open*' and correspond to grand-canonical ensembles. In principle, systems producing mechanical work should also be taken into account, but they are not considered here because of the assumption that no reaction-producing gas is occurring in the solutions studied.

The specification of the internal conditions consists in the definition of the type of the internal distribution of enthalpies and number of states. In defining distributions and ensembles, the term enthalpy is considered analogous to energy. There is no loss of generality by using enthalpy instead of internal energy in systems at constant pressure.

A distribution with constant enthalpy has null dispersion around the mean but not so around the average entropy or dilution. Such a system is called *non-reacting*.

If the enthalpy of the cells is not constant, and the molecules in the cells are those of an undiluted pure compound, then the dispersion around the mean enthalpy is not null. The system is composed of one component and is considered, at least in the ideal case, *non-reacting*.

The distribution might show distinct peaks whose difference in enthalpy is significantly higher than the

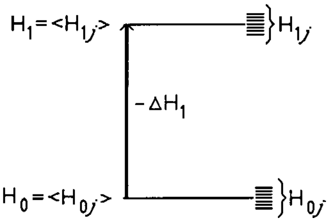

Fig. 1. Intralevel $\langle H_{0j} \rangle$ and interlevel ΔH_1 enthalpy at the molar level.

dispersion around the single peaks. The system is *reacting*. In fact, any *reacting* system is assumed to possess, on the molar scale, enthalpy levels H_i well separated and distinct from translational, vibrational, and rotational enthalpy sublevels $H_{i,j}$ (Fig. 1).

The existence of at least two separate enthalpy levels corresponds to the existence of at least two different species or clusters that can be transformed into one another by chemical reaction. ‘Saturation’ is the completion of the reaction. At the microscopic level of description, the molecular enthalpy is designated by the greek letter η.

3. Non-reacting ensembles

The following definitions characterize the statistical ensembles (Table 1):

3.1. Microcanonical ensemble

This ensemble is the statistical thermodynamic representation of a *non-reacting*, one-level system closed to both exchange of heat and to exchange of matter or isolated system.

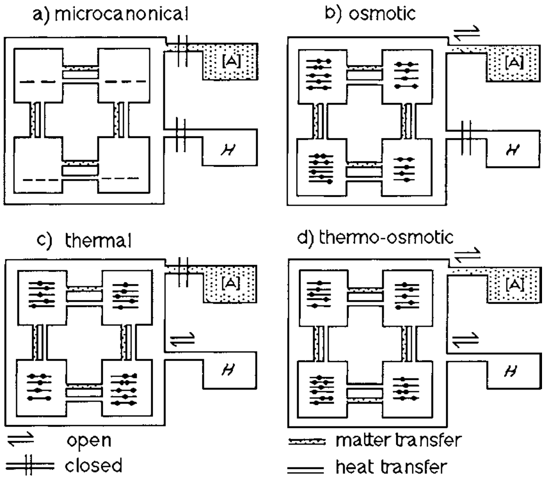

Fig. 2. Non-reacting ensembles with heat (H) and matter ([A]) reservoires: (a) *Microcanonical* nre_μ; (b) *osmotic* nre_o; (c) *thermeal* nre_t; and (d) *thermo-osmotic* $nre_{t,o}$. sublevels *j* of single mean enthalpy level.

The *microcanonical* ensemble is given the symbol

$$nre_\mu = nre_{S([A]),H(T),P,T,} \tag{1}$$

The subscripts indicate the constant parameters. The ensemble has definite and constant properties. The ensemble can be schematically represented in Fig. 2(a).

3.2. Osmotic non-reacting ensemble

This ensemble is the statistical thermodynamic representation of a *non-reacting*, one-level system *closed* to exchange of heat and *open* to exchange of solvent or solute. The *osmotic non-reacting* ensemble is given the symbol

$$nre_o = nre(S([A]))_{H(T),P,T,} \tag{2}$$

Table 1
Statistical ensembles (non-reacting)

Ensemble	Boundary conditions [a]		
	P	[A]	*H*
microcanonical $nre_\mu = nre(S)_{[A],H(T),P,T}$	i	c	c
osmotic $nre_o = nre(S([A]))_{H(T),P,T}$	i	o	c
thermal $nre_t = nre(T,H(T),S(T))_{[A],P}$	i	c	o
thermo-osmotic $nre_{t,o} = nre(T,H(T,[A]),S(T,[A]))_P$	i	o	o

[a] i, isobaric; c, closed; o, open (c,c) or (c,o), canonical; (o,c) or (o,o), grand canonical.

where variable (in parenthesis) and invariant (subscript) thermodynamic parameters are indicated. The ensemble changes its properties by changing the entropy, $S([\mathrm{A}])$, i.e. by dilution or concentration of the component A. The temperature, T, the pressure, P, and the enthalpy, $H(T)$ are constant. The scheme of the ensemble is drawn in Fig. 2(b).

On the macroscopic scale, it might be represented by an inert component in a solution with null solution enthalpy, in contact with the solvent reservoir through a semipermeable membrane. This ensemble conforms to the first postulate of statistical thermodynamics which states that the properties of the whole system are the average of the properties of the many Gibbs cells composing the ensemble, at any one instant. This ensemble conforms also to the condition that any system is moving towards maximum probability that coincides with the maximum number of accessible cells.

The changing parameter of the solute A is the activity a_A or number density, in the ideal case. In real systems the concentration [A] in mol dm^{-3} is related to the activity by the activity coefficient $a_\mathrm{A}=f_\mathrm{A}[A]$. The reciprocal of a_A is the dilution d_A whose logarithm is proportional to the entropy of the solution according to the Boltzmann law.

Under standard unitary conditions at $a_\mathrm{A}=1$, the ensemble is the standard osmotic ensemble

$$nre_\mathrm{o}^{\phi} = nre(S(a_\mathrm{A} = 1))_{H(T),P,T} \tag{3}$$

which is the ensemble whose chemical potential is $\mu=\mu^{\phi}$.

The solution enthalpy is assumed to be null. This assumption is required by the condition that the system is at constant temperature and adiabatic. This model is analogous to the microcanonical ensemble of statistical mechanisms and is useful to isolate the mixing entropy as the only thermodynamic variable contributing to the free energy of the system.

3.3. Thermal non-reacting ensemble

This ensemble is the statistical thermodynamic representation of a *non-reacting* system *open* to exchange of heat and *closed* to exchange of matter (canonical ensemble). The *thermal* ensemble is given the symbol

$$nre_\mathrm{t} = nre(T, H(T), S(T))_{[\mathrm{A}],P} \tag{4}$$

The ensemble changes its properties by changing the entropy $S(T)$ and the enthalpy $H(T)$ by heating or cooling the compound. The activity of the species [A] is constant and the pressure P, constant as well. On the macroscopic scale, it might represent a pure compound in one single phase or a non-reacting solute at constant concentration. This ensemble (Fig. 2(c)) conforms to the first postulate of statistical thermodynamics which states that the system can be represented in the course of the time by successive states of the cells which are the same as that used to represent in space and at one instant the whole system.

3.4. Thermo-osmotic non-reacting ensemble

Each single species which is part of a *reacting system* is characterized by the properties of a *thermal* ensemble open to heat exchange (dependence of entropy upon the temperature) with an *osmotic* ensemble open to matter exchange (dependence of entropy upon the dilution). The species taken per se is *non-reacting*. The *thermo-osmotic non-reacting* ensemble is given the symbol

$$nre_\mathrm{t,o} = nre(T, H(T, [\mathrm{A}]), S(T, [\mathrm{A}]))_P \tag{5}$$

and represented in Fig. 2(d).

In $nre_\mathrm{t,o}$, the entropy can change either by changing T or [A]. An equivalence between them can be found (see Appendix A). This equivalence can be thought of as the consequence of the combination of the first and second postulate of statistical thermodynamics. The probability of finding the species in the unit cell of the Gibbs space changes either as we change the number of accessible cells over which average is calculated wither as we change the speed whereby the species is moving from one to another cell. The equivalence of the changes of the entropy functions $S(T)$ and $S([A])$ is called *thermal equivalent dilution*.

4. Reacting ensembles

The *reacting* ensemble is the statistical thermodynamic representation of a *reacting* system *open* the exchange of heat, and/or matter, and/or electrons

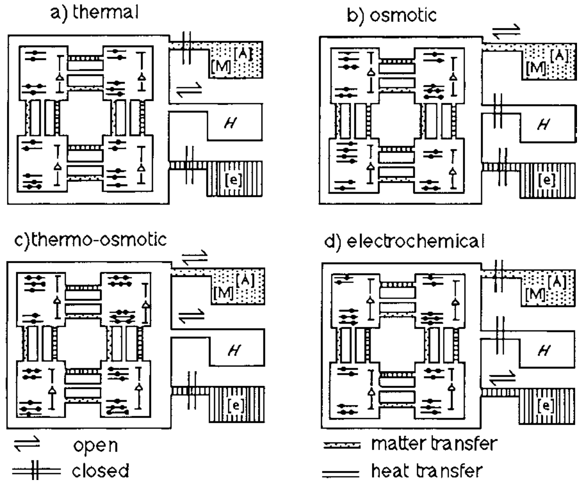

Fig. 3. Reacting ensembles with heat (H), matter ([A],[M]) and electricity ([e]) reservoirs: (a) *thermal* re_t; (b) *osmotic* re_o; (c) *thermo-osmotic* $re_{t,o}$; and (d) electrochemical re_e. Δ separation of different mean enthalpy levels occupied by different species.

(Fig. 3). The existence of the enthalpy difference written as Δ in the diagrams is a necessary condition for having an *re*.

The *re* changes its properties as the function of the reaction entropy $\Delta_r S$ and the reaction enthalpy, $\Delta_r H$. The variables to take into account are the temperature T, the free ligand concentration [A], and free receptor concentration [M]. Note that according to the reacting character of the ensemble the matter reservoir contains both, free A and free M. Strictly speaking even one of the component [A] or [M] alone is sufficient if the system undergoes a dimerization or polymerisation process. Changes are produced either by heating or cooling the system and/or by addition or subtraction of the component A and/or M. Linked changes of temperature are produced by addition of substances due to the reaction enthalpy or changes in composition are produced by changing the temperature which displaces the equilibrium. The pressure P is kept constant.

On the macroscopic scale, it might correspond to the solution of a receptor, M in equilibrium with a ligand A forming complexes MA_i.

The *re* are distinguished into *thermal*, *osmotic*, *thermo-osmotic*, *electrochemical*, *electro-osmotic*, *electro-thermal*, *electro-thermo-osmotic* ensembles depending on the type of reaction and of exchange with the surroundings (Table 2).

4.1. Thermal reacting ensemble

It is characterized by the absence of exchange of matter with the surroundings (Fig. 3(a)) and is, therefore, a canonical ensemble. It is contained in a diathermal vessel. The change of the temperature of the surrounding bath produces inside the system a change in the equilibrium composition accompanied by heat effects which are added up to or subtracted from the heat supplied

$$re_t = re\{T, H(\Delta_r H, T, [\mathrm{A}], [\mathrm{M}]), S(\Delta_r S, T, [\mathrm{A}], \times [\mathrm{M}])\}_{\mathrm{P,M_{tot},A_{tot}}} \tag{6}$$

Table 2
Statistical ensembles (reacting)

Ensemble	Boundary conditions		
	P	[A],[M]	Heat
$re_o\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}]), S(\Delta_r S,T,[\mathrm{A}],[\mathrm{M}])\}_{\mathrm{P}}$	i	o	c
$re_t\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}]), S(\Delta_r S, T, [\mathrm{A}], [\mathrm{M}])\}_{P,T,\mathrm{M_{tot}},\mathrm{A_{tot}}}$	i	c	o
$re_t\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}]), S(\Delta_r S, T, [\mathrm{A}], [\mathrm{M}])\}_{P,T,\mathrm{M_{tot}},\mathrm{A_{tot}}}$	i	c	o(is)
$re_{t,o}\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}]), S(\Delta_r S,T,[\mathrm{A}],[\mathrm{M}])\}_{\mathrm{P}}$	i	o	o
$re_{t,o}\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}]), S(\Delta_r S,T,[\mathrm{A}],[\mathrm{M}])\}_{\mathrm{P}}$	i	o	o(is)
$re_e\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}],[\mathrm{e}]), S(\Delta_r S, T, [\mathrm{A}], [\mathrm{M}][\mathrm{e}])_{P,\mathrm{M_{tot}},\mathrm{A_{tot}}}$	i	c	c
$re_{t,e}\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}],[\mathrm{e}]), S(\Delta_r S, T, [\mathrm{A}], [\mathrm{M}][\mathrm{e}])\}_{P,M_{\mathrm{tot}},\mathrm{A_{tot}}}$	i	c	o
$re_{e,o}\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}]),[\mathrm{e}]), S(\Delta_r S,T,[\mathrm{A}],[\mathrm{M}][\mathrm{e}])\}_{\mathrm{P}}$	i	o	c
$re_{t,e,o}\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}],[\mathrm{e}]), S(\Delta_r S,T,[\mathrm{A}],[\mathrm{M}][\mathrm{e}])\}_{\mathrm{P}}$	i	o	o
$re_{t,e,o}\{T,H(\Delta_r H,T,[\mathrm{A}],[\mathrm{M}],[\mathrm{e}]), S(\Delta_r S,T,[\mathrm{A}],[\mathrm{M}][\mathrm{e}])\}_{\mathrm{P,T}}$	i	o	o(is)

[a] i, isobaric; c, closed; o, open; o(is), open (isothermal); (c,c) or (c,o), canonical; and (o,c) or (o,o), grand canonical.

Easily interpretable results can be obtained by putting the system in isothermal equilibrium with the external heat sink, in an isothermal calorimeter. Alternatively, the system can be followed in a differential scanning calorimeter by determining the heat supplied to the reaction.

4.2. Osmotic reacting ensemble

This ensemble is characterized by open exchange of matter with the surroundings (Fig. 3(b)). The exchange of substance [A] and/or [M] with the outside can produce also internal heat effects because of the displacement of the endo- or exothermic reaction

$$re_{\mathrm{o}} = re\{T, H(\Delta_{\mathrm{r}}H, T, [\mathrm{A}], [\mathrm{M}]), \\ \times S(\Delta_{\mathrm{r}}S, T, [\mathrm{A}], [\mathrm{M}])\}_{\mathrm{P}} \tag{7}$$

The system is contained in a thermally isolated vessel and the heat produced or consumed by the displacement of the reaction can generate a change of temperature. The enthalpy H and entropy S are the function of the temperature at which the system is brought by the internal heat supply, of the reaction enthalpy $\Delta_{\mathrm{r}}H$ and reaction entropy $\Delta_{\mathrm{r}}S$ and of the change in free [A] and [M].

4.3. Thermo-osmotic reacting ensemble

It is characterized by open exchange of both heat and matter (Fig. 3(c)) and is therefore a grand canonical ensemble

$$re_{\mathrm{t,o}} = re\{T, H(\Delta_{\mathrm{r}}H, T, [\mathrm{A}], [\mathrm{M}]), \\ \times S(\Delta_{\mathrm{r}}S, T, [\mathrm{A}], [\mathrm{M}])\}_{\mathrm{P}} \tag{8}$$

The most easily interpretable results can be obtained when the exchange of heat is realized in isothermal conditions. In such a case, the whole heat produced or absorbed by the reaction is transformed into an equivalent entropic factor which is measured together with the intrinsic entropy as total change or equilibrium constant. A typical example of this ensemble is an isothermal reaction calorimeter. In this case also, the enthalpy content depends on the molar reaction enthalpy $\Delta_{\mathrm{r}}H$ and on the amount of the reaction displacement.

4.4. Electrochemical reacting ensemble

It is characterized by closed exchange of both heat and matter but is electrically connected to an external electrical circuit (Fig. 3(d)). The internal reaction enthalpy is due to an oxo-reduction reaction and it sensible to the exchange of electrons that is guaranteed by the conducting wires

$$re_{\mathrm{e}} = re\{T, H(\Delta_{\mathrm{r}}H, T, [M], [\mathrm{A}], [\mathrm{e}]), \\ \times S(\Delta_{\mathrm{r}}S, T, [\mathrm{A}], [\mathrm{M}][\mathrm{e}])_{\mathrm{P,M_{tot},A_{tot}}} \tag{9}$$

Even in this case, the existence of a free energy difference inside is the origin of the *reaction* energy. The supply of electrons can produce displacement of the reaction, thus producing heat effects which change the temperature inside the system.

4.5. Thermo-electrochemical reacting ensemble

It is characterized by closed exchange of matter but by open exchange of electrons. It is therefore a grand canonical ensemble. The internal reaction is an oxo-reduction reaction and it sensible to the exchange of electrons that is guaranteed by conducting wires

$$re_{\mathrm{t,e}} = re\{T, H(\Delta_{\mathrm{r}}H, T, [\mathrm{A}], [\mathrm{M}], [\mathrm{e}]), \\ \times S(\Delta_{\mathrm{r}}S, T, [\mathrm{A}], [\mathrm{M}][\mathrm{e}]\}_{\mathrm{P,M_{tot},A_{tot}}} \tag{10}$$

The electrons can produce displacements in the reaction. The reaction is studied when the exchange of heat is realized in isothermal conditions. In such a case, the whole heat produced or absorbed by the reaction is transformed into an equivalent entropic factor which is measured together with the intrinsic entropy as total entropy change.

4.6. Electro-osmotic reacting ensemble

It is characterized by open exchange of matter and electrons through connections with the surroundings while the walls are thermally isolated. The internal oxo-reduction reaction is accompanied by one or more reactions coupled to the exchange of electrons

$$re_{\mathrm{e,o}} = re\{T, H(\Delta_{\mathrm{r}}H, T, [\mathrm{A}], [\mathrm{M}], [\mathrm{e}]), \\ \times S(\Delta_{\mathrm{r}}S, T, [\mathrm{A}], [\mathrm{M}][\mathrm{e}])\}_{\mathrm{P}} \tag{11}$$

The internal reaction can produce heat which is not transmitted outside because of the athermal walls.

Consequently, a change of temperature can be detected in the system.

4.7. Thermo-electro-osmotic reacting ensemble

It is characterized by open exchange of matter, heat and electrons. The internal oxo-reduction is accompanied by one or more reactions coupled to the exchange of electrons

$$re_{\mathrm{t,e,o}} = re\{T, H(\Delta_{\mathrm{r}}H, T, [\mathrm{A}], [\mathrm{M}], [\mathrm{e}]), \times S(\Delta_{\mathrm{r}}S, T, [\mathrm{A}], [\mathrm{M}][\mathrm{e}])\}_{\mathrm{P}} \qquad (12)$$

The transformations in the system can be studied when the exchange of heat is realized in isothermal conditions. Typical examples are electrochemical chains employed in potentiometric apparatus or coupled reactions taking place isothermally in biological cells or membranes.

5. Fluctuations

The classification of ensembles as *reacting* and *non-reacting* is substantially confirmed by the types of fluctuations of thermodynamic variables presented by the two classes. The fluctuation of a thermodynamic variable can be identified with the change of entropy produced by a change of that variable. We can consider the fluctuations of *non-reacting* ensembles nre_{o}, nre_{t}, and reacting ensembles re_{o}, re_{t}, presented by the two types of ensembles.

The variance indicates the buffer capacity or maximum change of the system with respect to change in the corresponding variable. The fluctuating variables for each ensemble are those for which the system is open to the surroundings (Table 3). The fluctuation or variance of the function is calculated by means of the derivative of entropy with respect to the variable. The values in the Table 3 are referred to macroscopic ensembles of mol (R is used instead of k_{B} as the reference quantity). Correspondingly capital letters are used for molar thermodynamic functions. This scale corresponds to the experimental scale of heat capacity and buffer capacity.

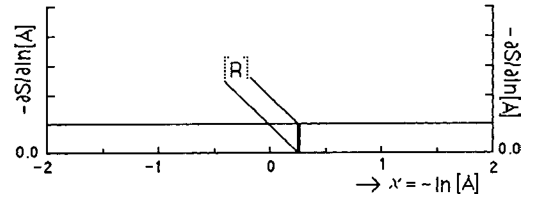

Fig. 4. Entropy fluctuations in *osmotic non-reacting ensemble* nre_{o}. The diagram can be comapred with the spectrum of a colorless non absorbing compund.

In *osmotic non-reacting* ensembles nre_{o}, there is no dispersion of enthalpy whatsoever, because the enthalpy is assumed constant.

In nre_{o}, the dispersion or variance for entropy with respect a change of the concentration is that shown in Fig. 4. The variance of entropy at the microscopic levels is entirely due to variability of ln[A]. The derivative $(-\partial(S)/\partial \ln\,[\mathrm{A}])$ equals the gas constant R at the molar level.

In nre_{t}, the dispersion of entropy is given by the isobaric heat capacity C_{p}, as shown in Fig. 5. The variance increases smoothly as the temperature increases if the process takes place without change of phase. If a change of phase occurs the system

Table 3
Fluctuations

Non-reacting ensembles		
Osmotic	nre_{o}	$\mathrm{var}(-H/RT)=0$ $\mathrm{var}(S/R)=\partial(S/R)/\partial \ln(1/[\mathrm{A}])=1$ $\mathrm{var}(-G/RT)=\partial(-G/RT)/\partial \ln(1/[\mathrm{A}])=1$
Thermal	nre_{t}	$\mathrm{var}(-H/R)=C_{\mathrm{p}}/R$ $\mathrm{var}(S/R)=\partial(S/R)/\partial \ln T=C_{\mathrm{p}}/R$
Reacting ensembles		
Osmotic	re_{o}	$\mathrm{var}(-\Delta G/RT)=\partial\bar{n}/\partial \ln[A] = \partial^2\bar{n}/\partial \ln[\mathrm{A}]^2$
Thermal	re_{t}	$\mathrm{var}(-\Delta \mathrm{G/RT})=\partial(-\Delta \mathrm{G/RT})/\{\partial \ln[\mathrm{A}]\partial \ln\text{~}T)$ $=(\Delta C_{\mathrm{p,app}}/R)=\alpha(1-\alpha)(-\Delta H/RT+C_{\mathrm{p,[A]}}/R)$

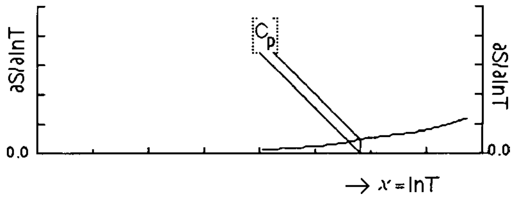

Fig. 5. Entropy fluctuation in *termal non-reacting* ensemble nre_tC_p is molar isobaric heat capacity. The diagram can be compared with the spectrum of a colourless non-absorbing compound.

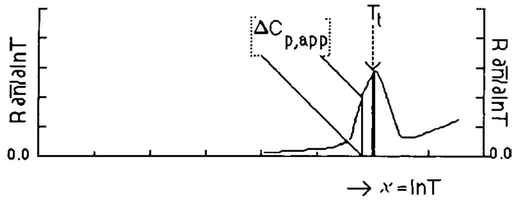

Fig. 7. Fluctuations in *thermal reacting* ensemble re_t. The diagram can be compared with the band spectrum of an absorbing compound (one band).

becomes *reacting*. The changes in enthalpy and changes in entropy are indistinguishable.

The fluctuations or variances in *reacting* ensembles show features completely different from those of the corresponding *non-reacting* ensembles. The dramatically different behaviour is the main justification of the distinction of systems into the two classes of *non-reacting* and *reacting*. In *osmotic reacting* ensemble re_o, the fluctuations are produced by the change in the concentration of the ligand A is shown in Fig. 6. The peculiarity of this diagram is that in correspondence with the reaction M+A=MA or MA+A=MA$_2$ the variance increases up to the maximum of buffer capacity at half way of the reaction. At the microscopic molecular scale the variance is numerically the same even it is written as the function of k_B and log [a] where [a] is concentration in molecules per unit cell. The fluctuations in *osmotic* re_o with respect to the variable ln [A] are determined by the second derivative of free energy with respect to ln [A] or the first derivative of the formation function $\bar{n}$ with respect to the same variable. The variance is coincident with the buffer capacity. There are two maxima of buffer capacity in coincidence with ln [A]=$-\Delta G_1/RT$ and $-\Delta G_2/RT$, respectively. ΔG_1 is the free energy of formation of the complex MA and ΔG_2 the stepwise free energy of formation of the complex MA$_2$.

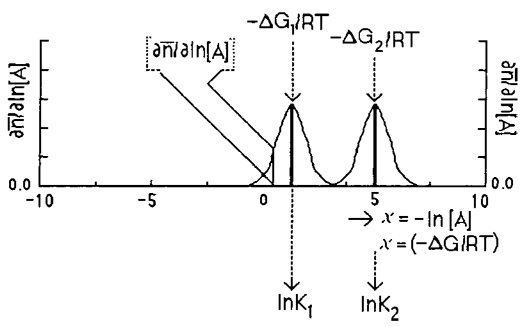

Fig. 6. Flucutations in *osmotic reacting* ensemble re_o. Two stepwise equilibrium constants are assumed with standard free energy $\Delta G_1/RT$ and $\Delta G_2/RT$, respectively. The diagram can be compared with the band spectrum of an absorbing compound (two bands).

In thermal reacting ensembles re_t, the variance is produced by the advancement of the reaction taking place at the transformation point T_t by means of the heat supplied to the system or even, in endothermic reactions subtracted from the system (Fig. 7). The variability of the reacting ensemble depends on the heat produced inside the system. The advancement of the reaction is measured by the molar fraction α, of the species produced by the change of temperature. The dispersion of values, or in other words the sensitivity of the system is given at the molar level by the buffer capacity ($\alpha(1-\alpha)$) multiplied by a weighting function which depends on the reaction enthalpy and on the heat capacity of the ligand A.

The connection between variance and properties of *reacting* and *non-reacting* ensembles can be better explained if one compares the diagrams of Figs. 4 and 5 with the void spectrum of a colourless compound with no inter-level transition. On the contrary, Figs. 6 and 7 can be compared with a spectrum of a solution containing two and one absorbtion band, respectively. The absorption of light takes place only if there are two distinct energy levels for the transition.

6. Conclusion

The classification of ensembles as *reacting* and *non-reacting* is useful to characterize the properties of the thermodynamic systems.

The existence of a definite enthalpy difference $\Delta_r H$ in the distribution is the necessary and sufficient condition to write an equilibrium constant. The *reacting* and *non-reacting* ensembles differ one another because one cannot write any equilibrium constant for *non-reacting* ensembles. The equilibrium constant is a typical statistical parameter of the reaction.

The ultimate purpose of statistical thermodynamics is to express the interactions between components by sum of molecular potentials. The distinction between *non-reacting* and *reacting* ensembles can be referred to the existence in the latter of an appropriate well-separated couple of potentials curves. The existence of a definite enthalpy between the minimum of two pair curves is the necessary condition to distinguish a *reacting* from a *non-reacting* ensemble. The definite enthalpy difference is necessary to have an equilibria between different chemical species in the experimental system.

The properties of the various ensembles are represented mathematically on the macroscopic scale by partition functions whose derivatives correspond to the observables. The partition functions are probability distributions and are geometrically represented in the 'probability space' whereas the first moments of the probability distributions are represented in the 'thermodynamics' space [10,11].

The distinction of *reacting* and *non-reacting* ensembles yields corresponding differences in the thermodynamics and probability domain which are fundamental to show the connections between the properties of the single representative element of the ensemble and the experimental properties of the whole system.

The equilibrium constant for grand canonical ensemble indicates that the same system, when out of equilibrium, has a potential energy to do chemical work to be treated by non-equilibrium thermodynamics (NET). On the other hand, the possibility for canonical ensembles to exchange matter and heat with the surroundings shows that canonical ensembles also can be active for NET, in the form nre_o, nre_t, and $nre_{t,o}$ to produce osmotic and/or thermal work.

7. List of symbol

nre	*non-reacting* ensemble
re	*reacting* ensemble
nre_μ	*microcanonical non-reacting* ensemble
nre_o	*osmotic non-reacting* ensemble
$S([A])$	entropy dependent upon dilution or concentration of the component A
T	temperature
P	pressure
$H(T)$	enthalpy as the function of the temperature
$S(T)$	entropy as the function of the temperature
$[A],[M],[MA_i]$	concentration or number density of species A, M, MA_i.
a_A	activity of species A
d_A	dilution of species A
f_A	activity of coefficient of the species A
nre_t	*thermal non-reacting* ensemble
$nre_{t,o}$	*thermo-osmotic non-reacting* ensemble
TED	*thermal equivalent* dilution
re_o	osmotic reacting ensemble
$\Delta_r S$	reaction entropy
$\Delta_r H$	reaction enthalpy
$\Delta_r G$	reaction free-energy
re_t	*thermal reacting* ensemble
M_{tot}, A_{tot}	total amount of M or A in any species
$re_{t,o}$	*thermo-osmotic reacting* ensemble
re_e	*electrochemical reacting* ensemble
$re_{t,e}$	*thermal-electrochemical reacting* ensemble
$re_{e,o}$	*electro-osmotic reacting* ensemble
$re_{t,e,o}$	*thermo-electro osmotic reacting* ensemble
k_B	Boltzmann constant
[a]	concentration of species A at the molecular level
R	gas constant
η	enthalpy at the molecular level
α	reaction fraction
nre_o^ϕ	standard osmotic non-reacting ensemble
μ	chemical potential
μ^ϕ	standard chemical potential
Δ	separation of different mean enthalpy levels
[e]	electron activity

C_p	isobaric heat activity
var	variance
η	enthalpy at molecular level
$\Delta_r G$	reaction free energy
G	free energy
H	enthalpy
S	entropy
s	entropy at molecular level
$\Delta C_{p,app}$	apparent isobaric
$\bar{n}$	formation function
T_t	temperature of transition
$[A]_{vir}$	virtual concentration
H_i	enthalpy of level *i*
$H_{i,j}$	enthalpy of sublevel *i,j*
I	ionic strength

Acknowledgements

This work has been supported by the Project 'Termodinamica dei complessi' of the Italian Ministry for University and for Scientific and Technological Research (MURST).

Appendix A

Activity coefficients

The entropy S of an open non-reacting ensemble is a function of temperature (T) and activity (or concentration [A], by assuming unitary activity coefficient as in ideal systems)

$$S = S(T, [A]) \tag{A.1}$$

which can be differentiated with respect to the variables

$$dS = \{\partial S/\partial \ln T\}_{[A]} d\ln T + \{\partial S/\partial \ln(1/[A])\}_T d\ln(1/[A]) \tag{A.2}$$

The partial derivatives can be identified as

$$dS(T) = C_p d\ln T \tag{A.3}$$

and

$$dS([A]) = R d\ln(1/[A]) \tag{A.4}$$

respectively and substituted in Eq. (A.2).This yields

$$dS = C_p d\ln T + R d\ln(1/[A]) \tag{A.5}$$

If one increases the temperature, then the entropy of the system increases because $C_p d\ln T$ is a positive quantity. The entropy can be restored to its primitive value ($dS=0$,S constant) by decreasing the entropy by an amount $-R d\ln(1/[A]) = -C_p d\ln T$, what can be done by increasing the concentration [A]. Therefore, the virtual change of concentration $[A]_{vir}$

$$C_p d\ln T = R d\ln(1/[A]_{vir}) \tag{A.6}$$

is the thermal equivalent dilution.

The same arguments can be applied to the activity coefficients. The entropy S of an open *non-reacting* ensemble at constant T is a function of ionic strength I, i.e. activity coefficient f_A and concentration [A]

$$S = S(f_A, [A]) \tag{A.7}$$

which can be differentiated with respect to the variables

$$dS = \{\partial S/\partial \ln f_A\}_{[A]} d\ln f_A + \{\partial S/\partial \ln[A]\}_I d\ln[A] \tag{A.8}$$

which can be written by recalling the equality $\{\partial S/\partial \ln f_A\}_{[A]} = \{\partial S/\partial \ln[A]\}_I = -R$

$$dS = -R d\ln f_A - R d\ln[A] \tag{A.9}$$

If the activity coefficient is changed by changing the ionic strength I then the same change of entropy can be compensated by increasing or decreasing the concentration. The latter change of concentration with the sign of the logarithm changed

$$R\, d\ln f_A = -R\, d\ln(1/[A]_{vir}) \tag{A.10}$$

is the electrostatic equivalent dilution.

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